

## Time-Domain Calculation of Microstrip Components and the Curve-Fitting of Numerical Results

Xiaolei Zhang and Kenneth K. Mei

Department of Electrical Engineering and Computer Sciences  
University of California, Berkeley, CA 94720

### Abstract

The Time-Domain Finite Difference method combined with Fourier transform techniques has recently been shown to be a very effective approach in modeling the dispersive characteristics of microstrip components[1-3]. The present research further investigate the problems associated with the generation of design data over a larger substrate and line parameter range and the curve-fitting of the numerical results for CAD purpose.

### Introduction

The numerical calculations of the dispersive characteristics of the microstrip related structures are mostly done in frequency domain[4-6]. The recent research on the time-domain simulation of a Gaussian pulse propagation in the microstrip structure and the extraction of the frequency-domain design data via the Fourier transform of the time-domain pulse response has provided another independent approach for the microstrip component modeling[1-3]. This approach has been shown to be able to characterize a large variety of microstrip transmission structures and microstrip discontinuities over wide frequency range. The current research efforts are towards the calculation of the design data for wider range of microstrip line and substrate parameters, and to curve-fit the obtained numerical results into closed form formulas to be incorporated into CAD packages.

### Time-Domain Finite Difference Approach

The calculation of the time-domain fields is done using the Time-Domain Finite Difference method[7]. In applying this method, the two Maxwell's curl equations are discretized both in time and space, and the field values on the nodal points of the space-time mesh are

calculated in a leap-frog time marching manner once the initial and boundary conditions are specified. The artificial absorbing boundary condition is use on those outer boundaries of the computation domain where a transparent or non-reflection boundary condition is needed [1-2].

Fig.1 shows a typical finite difference computation domain for microstrip open-end discontinuity used here as an example (because of the symmetry of the problem, only half of the structure is considered). Fig.2 shows the calculated time domain incident and reflected pulse waveform for microstrip open-end. The ratio of the Fourier transforms of the reflected and the incident pulse will give the S-parameter of the structure. The details of this calculation can be found in [2-3].

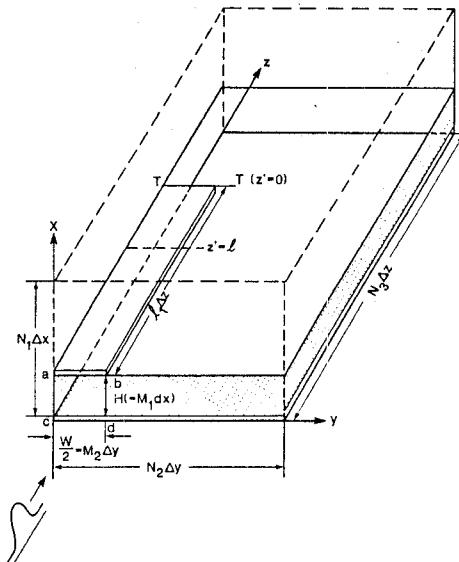


Figure 1. Microstrip Open-End Terminations (  $W=H=0.6\text{mm}$ ,  $\epsilon_r = 9.6$ ,  $\Delta h=H/10$ ,  $\Delta t=0.515 \Delta h/c$ ,  $t_0 = 350 \Delta t$ ,  $T = 40 \Delta t$  )

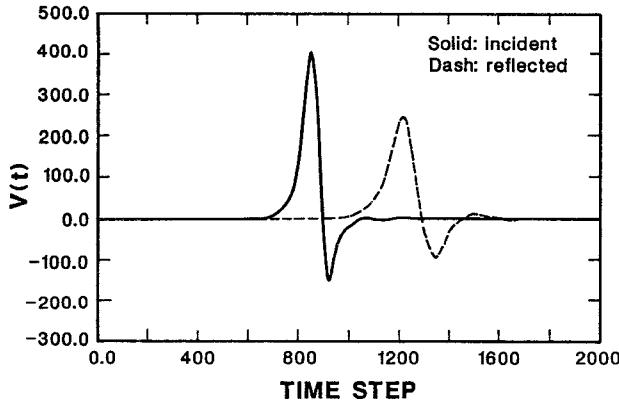


Figure 2. Gaussian Pulse Propagation and Reflection in the Microstrip Open-End Structure : Voltage Underneath the Center of the Microstrip

The Time-Domain Finite Difference method has been demonstrated to be able to model various microstrip transmission structures and microstrip discontinuities over wide frequency range[1-3]. But the previous calculation was done only for a limited variety of dielectric substrate material and for characteristic impedance of the line near  $50\text{ ohm}$  ( which usually means that the microstrip line width  $w$  and height  $h$  are of comparable size ). To generate enough data for CAD purpose a larger range of parameters of the microstrip must be covered. However, the Time-Domain Finite Difference method has difficulties when dealing with structures where both large and small geometric dimensions are involved, like in the case of very wide (  $w/h$  very large ) and very thin (  $w/h$  very small ) microstrips. This is due to the fact that the finite difference mesh used must be fine enough to achieve good resolution of the smallest dimension in the problem, and at the same time the total region that the mesh covers must be large enough to enclose the whole structure, especially as in our problems it has to be made even larger to allow a certain distance for the unrealistic reflections from the imperfect treatment of the artificial absorbing boundaries to diminish or to come back sufficiently late[2], so as to keep the time-domain data from being contaminated. These two conditions together result in the requirement of a very large computer memory which in many cases is often not available.

This problem is currently being dealt with by two approaches: 1. By making use of the rectangular-shaped mesh rather than the square-shaped mesh. This allows the mesh length to take smaller value only in the direction where the geometric dimension of the structure has a small value. Caution must be taken here in choosing the time step  $dt$  accordingly such that the stability

criterion is always satisfied[2]. This approach, although extended the range of  $w/h$  we are capable of calculating, is still not applicable to the very large and very small  $w/h$  case, since the very distorted mesh increases the computation error, and the boundary reflections are also observed to be increased. 2. By calculating the limiting cases first, thus changing the problem ( sometimes ) into a two dimensional one. For example, although we can not calculate the data for very large  $w/h$ , but the  $w/h = \infty$  case is a very easy one to calculate. The other limit (  $w/h=0$  ) is much harder to fix and is still under investigation. These limiting case data give us some ideas for the range of changes of a variable with  $w/h$ .

### Curve fitting of Numerical Results

After we obtain the frequency domain data for a certain range of frequency and line parameters, the next thing we seek to find is a closed form formula which best fit the numerical results such that it can be incorporated into CAD packages.

The first and most important step in curve fitting the computed data is to assume a reasonable functional form with the coefficients to be determined from the fitting. The number of independent variables in our problems are usually three, i.e. the frequency  $f$ , the dielectric constant of the substrate  $\epsilon_r$ , and the strip width over height  $w/h$  ( or equivalently the DC characteristic impedance  $Z_0$  ). Since there are still no good formulas available to characterize the frequency dependence of various microstrip discontinuities, our first task is to look for simple and meaningful functional forms with three degrees of freedom (  $f$ ,  $\epsilon_r$  and  $w/h$  or  $Z_0$  ) from our knowledge of the computed results, the physics of the discontinuity problem, the scaling effect and the limiting situations. Efforts are made to have each of these formulas being able to be transformed to the form which is linear in terms of the unknown coefficients, thus can be fitted by the linear least square curve fitting program.

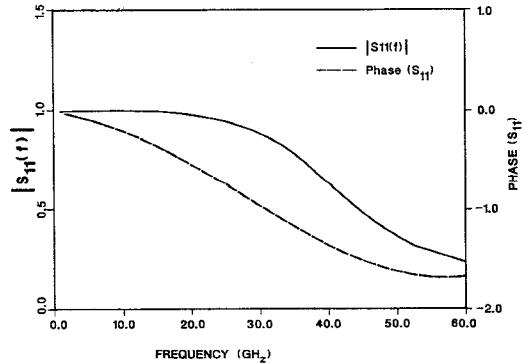


Figure 3. Frequency Dependant S-Parameter

The curve fitting of microstrip open-end results are done first as a testing case. The calculated magnitude and phase of the reflection coefficient of microstrip open-end is shown in Fig.3 for  $\epsilon_r = 9.6$  ( Alumina substrate ) and for  $w/h = 1$ . Inspired by the empirical formula for the effective dielectric constant of a uniform microstrip line obtained by Getsinger [9], and by Edwards and Owens [8], the functional form for the magnitude of the reflection coefficient  $S_{11}$  ( denoted by  $S$  in below ) is assumed to be

$$S(f) = S_\infty - \frac{S_\infty - S_0}{1 + P} \quad (1)$$

where  $P$  is a polynomial of the form

$$P = G_2 \left( \frac{f}{f_p} \right)^2 + G_3 \left( \frac{f}{f_p} \right)^3 + \dots + G_n \left( \frac{f}{f_p} \right)^n \quad (2)$$

and where

$$f_p = \frac{Z_0}{2\mu_0 h} \quad (3)$$

is the inflection frequency and  $G_2, G_3, \dots, G_n$  are coefficients to be determined from the curve fitting process,  $G_i = G_i(Z_0, \epsilon_r)$ . This formula have the following properties: 1)  $S = S_0$  when  $f = 0$ ; 2)  $dS/df = 0$  when  $f = 0$ ; 3)  $S$  approaches  $S_\infty$  as  $f$  approaches  $\infty$ ; 4) the scaling characteristics of the original solution is maintained, i.e. the result will be the same as long as  $f$  times  $h$  is the same. It is found that usually only  $G_2, G_3$  and  $G_4$  are needed to have a fairly good quality of fitting up to 30 Ghz.

The result of fitting for  $\epsilon_r = 9.6$ ,  $w/h = 1$  and the frequency range from zero to 30 Ghz is shown in Fig.4 as an example. Here  $S_0 = 1$ ,  $S_\infty = 0$ , and we obtain  $G_2 = 0.21$ ,  $G_3 = -0.77$  and  $G_4 = 0.78$ . The calculation result for the same substrate but with  $w/h = \infty$  is also shown in Fig. 4 to indicate of the range of change of  $S_{11}$ . The calculation and curve fitting for other values of  $\epsilon_r$  and  $w/h$  are still undergoing. The results for different  $\epsilon_r$  and  $w/h$  will allow us to obtain a representation of  $G$  as a function of  $\epsilon_r$  and  $w/h$ .

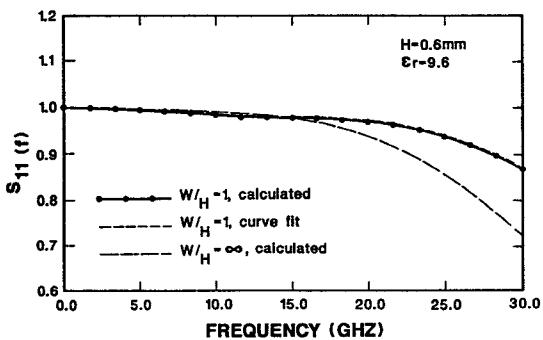


Figure 4. Curve Fitting of the Magnitude of  $S_{11}$ .

For curve fitting of the open-end results after 30 Ghz, it is found that the use of equivalent circuit is likely to ease the process of fitting. There are two types of equivalent circuits which can be used to characterize the open-end effect: the conventional parallel circuit for low frequency range and the here proposed series circuit for high frequency range (Fig.5). It is very obvious from the comparison of Fig.6 and Fig. 7 that at frequencies between 30 and 70 Ghz the series components have a much smoother variation with frequency than the parallel components.

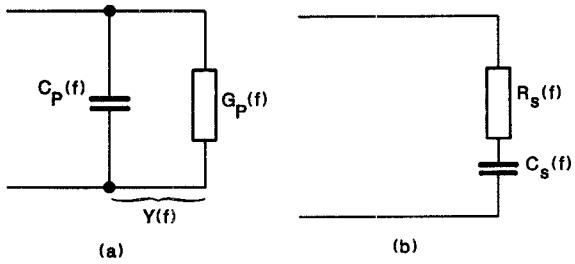


Figure 5. Equivalent Circuits of the Microstrip Open-End.

- (a) Equivalent Circuit consists of parallel components
- (b) Equivalent Circuit consists of series components

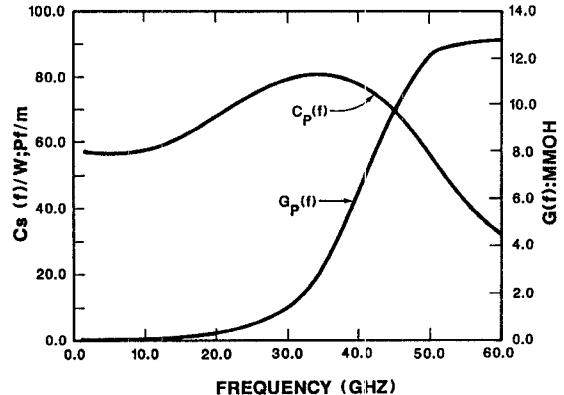


Figure 6. Frequency Dependence Equivalent Circuit Parameter  $C_p(f)/W$  and  $G_p(f)$

It is hoped that after a large amount of calculations are done for various microstrip discontinuities, closed-form formulas can be obtained to be used by circuit designers and to be incorporated in to the CAD packages for microwave integrated circuits and MMICs.

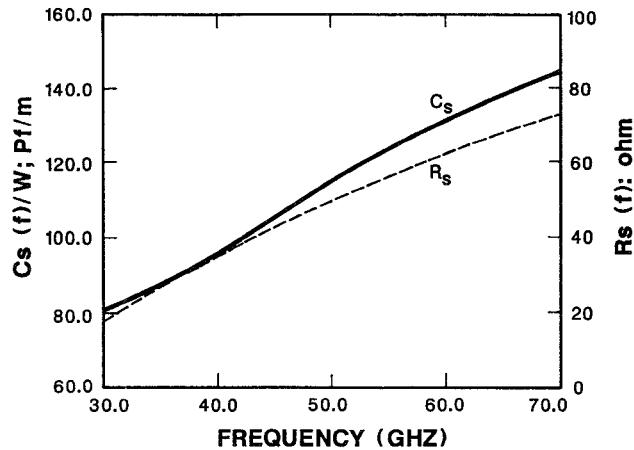


Figure 7. Frequency Dependence Equivalent Circuit Parameter  $C_s(f)/W$  and  $R_s(f)$

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